Lepton mass effects in weak CC single pion production

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Different approaches to take into account nonzero lepton mass in the Rein-Sehgal model are compared. Modification of the axial current due to pion pole term are included and it is shown that they lead to large reduction of antineutrino cross section and a change of the shape of $d\sigma/dQ^2$.

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The analysis of results of neutrino experiments is based on Monte Carlo generators of events. Because precise experimental data on inclusive and exclusive cross sections is still missing the generators rely on approximate models. In the 1 GeV neutrino energy region there is an important contribution from charged current (CC) and neutral current (NC) single pion production (SPP) channels. In most Monte Carlo (MC) codes [1] the dynamics responsible for SPP is described in the framework of the Rein-Sehgal (RS) model [2].

The RS model is an old construction based on the relativistic quark resonance FKR model [3]. It is known that in the case of electro-production its predictions are far away from the new precise experimental data but nevertheless the model is useful in describing neutrino interactions. There are several reasons why it is so. The model is suitable for MC applications as for SPP channels it provides a description of all degrees of freedom. The original RS paper provides a clear algorithm how to implement the model. Another advantage of the RS model is that it covers a large region in W ($W < 2$ GeV) in the kinematically allowed space. But the most important is that the predictions of the model agree with the existing data. The predictions for the integrated cross sections can be fine tuned to the data by modifying the value of the free parameter, an axial mass. An additional fine tuning can be done with a non-resonant background which must be added in some SPP channels. There are other more sophisticated and better founded approaches to describe SPP in the $\Delta(1232)$ resonance region like e.g. Lee-Sato model [4] but the RS model is still used and its performance must be regarded as an important topic of investigation.

The original RS approach adopts the assumption that leptonic mass $m = 0$. With this assumption all the computations in the model become easier. An approximate way to introduce $m \neq 0$ corrections is to consider them in the kinematics only. The procedure is to use the formula for $\frac{d^2\sigma}{dWdQ^2}$ as in the original RS model but to perform integration over the restricted kinematical region in the (W, Q^2) plane, the same as in the $m \neq 0$ case. This is the common way in which $m \neq 0$ effects are included in MC codes.

In recent years there is a growing number of indications that predictions of MC generators overestimate the cross section in the low Q^2 region. It is therefore important to check exactly the modifications introduced to the RS model by $m \neq 0$ effects and it is the subject of this paper. The modifications we consider are of two kinds. First we use the same hadronic current as in the RS model. The inclusion of $m \neq 0$ effects requires extra computation of hadronic weak current matrix elements. Our calculations are based on the observation that the operational structure of the components of the hadronic current $\mathcal{J}_0$ and $\mathcal{J}_3$ are identical. In the original RS paper the matrix elements of linear combination $\mathcal{J}_{\underline{0}} \equiv \mathcal{J}_0 + \frac{\nu_{res}}{q_{res}}\mathcal{J}_3$ are calculated. We use these results and by appropriate substitutions obtain matrix elements of $\mathcal{J}_0$ and $\mathcal{J}_3$ separately.

The second modification is more subtle and requires an inclusion of a new term in the axial current based on the PCAC arguments. The procedure how to modify the axial current was described in [5]:

$$\mathcal{J}^A_\mu \hookrightarrow \mathcal{J}^{A,mod}_\mu \equiv \mathcal{J}^A_\mu + q_\mu \frac{q^\nu \mathcal{J}^A_\nu}{m_\pi^2 + Q^2}. \tag{1}$$

The pion pole term does not contribute when the lepton mass is vanishing because the leptonic tensor $L_{\mu\nu}$ satisfies $L_{\mu\nu}q^\nu = -2m^2 k_\mu$. In the coordinate system in which $q^\mu = (\nu, 0, 0, q)$ the PCAC modifies only $\mathcal{J}_0$ and $\mathcal{J}_3$ components of the axial hadronic current and its inclusion can be again realized by substitutions in the original RS formulas. Some technical details about our calculations are contained in Appendix A.

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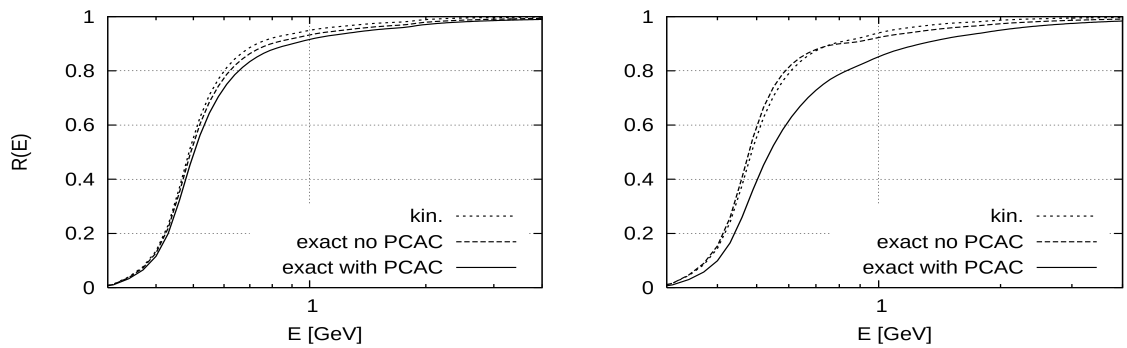

FIG. 1: Reduction of the total cross sections for reactions $\nu n \to \mu^- p \pi^0$ (left figure) and $\overline{\nu} p \to \mu^+ n \pi^0$ (right figure) due to $m \neq 0$. Three computations are compared: (i) kinematical approximation (dotted line) (ii) exact computation without pion pole contribution (dashed line) (iii) exact computations with pion pole contribution included (solid line). The cut on the invariant hadronic mass $W < 2$ GeV is imposed.

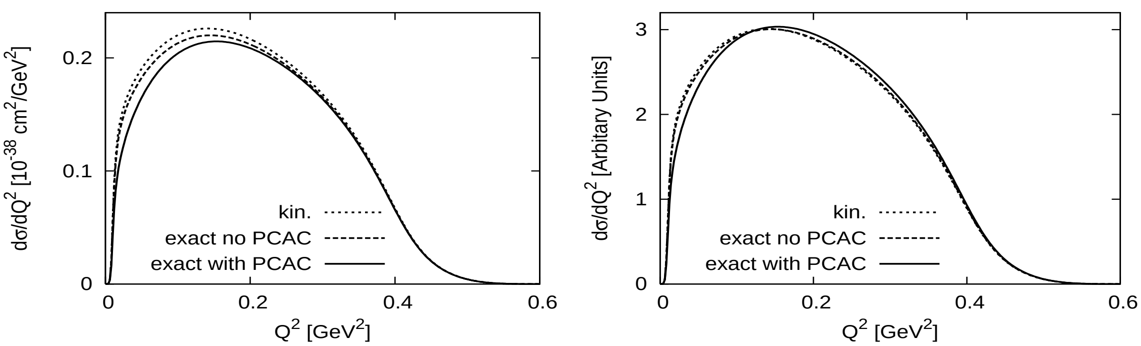

FIG. 2: The differential cross sections for reaction $\nu n \to \mu^- p \pi^0$ at $E = 700$ MeV calculated in three models: (i) kinematical approximation (dotted line) (ii) exact computation without pion pole contribution (dashed line) (iii) exact computations with pion pole contribution included (solid line). In the right panel the cross sections are normalized to the area under the curve. The cut on the invariant hadronic mass $W < 2$ GeV is imposed.

Few years ago Naumov's group extended the RS model to take into account the first part of $m \neq 0$ effects [6]. They expressed cross section by matrix elements of three currents: $\mathcal{J}_\pm$ (as in the RS model) and new component $\widetilde{\mathcal{J}}_{\underline{0}} \neq \mathcal{J}_{\underline{0}}$. It can be show that this part their calculations are equivalent to ours. However, there is an important difference: in [6] the pion pole contribution was not considered and we will see in the discussion that it is quite relevant.

In Fig. 1 we show the plots with relative modification of the cross section for $\nu n \to \mu^- p \pi^0$ and $\overline{\nu} p \to \mu^+ n \pi^0$ caused by $m \neq 0$. The functions we define are:

$$\mathcal{R}(E) = \frac{\sigma(E, m = m_\mu)}{\sigma(E, m = 0)}, \tag{2}$$

where $\sigma(E, m = m_\mu)$ is calculated in three different model:

(i) in *kinematical approximation* i.e. including $m \neq 0$ effects only in kinematics, as described above;

(ii) in exact computation but without pion pole contribution;

(iii) in the complete computations with pion pole contribution included.

In the case of neutrino-nucleon scattering all three approaches give rise to comparable results. At $E \sim 1$ GeV the differences between predictions for the total cross section is of order 5 % .

In the case of antineutrino-nucleon scattering the approaches (i) and (ii) give similar results while the exact computations (iii) with pion pole contribution introduce a significant reduction of the total cross section.

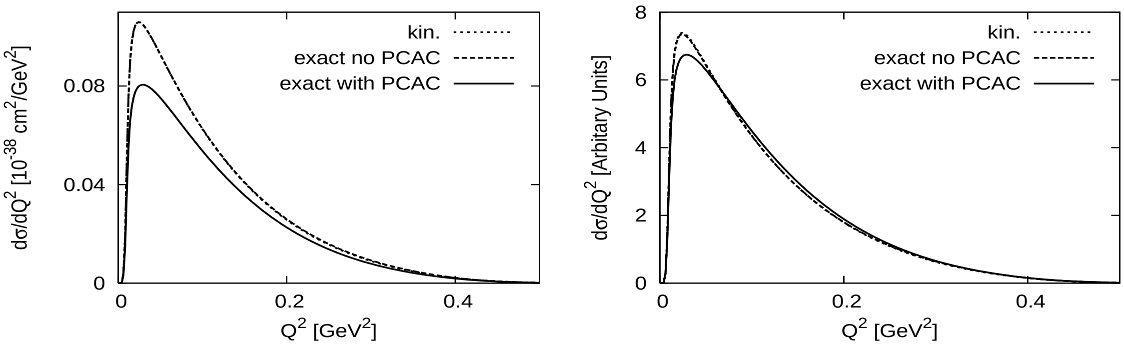

FIG. 3: The same as in Fig. 2 but for reaction $\overline{\nu}p \to \mu^+ n\pi^0$.

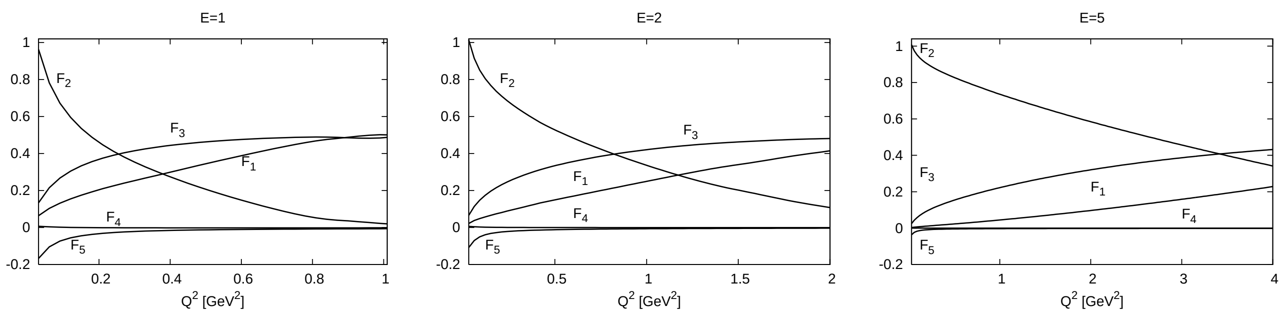

FIG. 4: The relative contributions from structure functions F_j in the differential cross section $d\sigma/dQ^2$ for reaction $\nu n \to \mu^- p\pi^0$ at neutrino energies 1, 2 and 3 GeV.

We verified that the same is true also for other neutrino and antineutrino induced SPP channels.

In the Fig. 2 we investigate $d\sigma/dQ^2$ (left panel) and its shape (right panel) for reaction $\nu p \to \mu^- p\pi^0$ at neutrino energy $E = 0.7$ GeV. Approaches (i) and (ii) give similar results. The contribution from pion pole visibly reduces the differential cross section for low Q^2 but its shape remains virtually unchanged. It is an important observation because in the experimental analysis the shape of $d\sigma/dQ^2$ is of major interests.

The situation becomes different in the case of antineutrino-nucleon scattering. In Fig. 3 we plot the same curves as in Fig. 2 but this time the reaction is $\overline{\nu}p \to \mu^+ n\pi^0$. The peak of the differential cross section in the approach (iii) is $\sim 25\%$ lower in the models (i) and (ii). What is more important also the shape of the differential cross section is changed and the peak is reduced by $\sim 10\%$.

The difference between neutrino and antineutrino scattering can be understood when the differential cross sections is expressed in terms of F_j, $j = 1, ..., 5$ structure functions (see Appendix B). Only F_4 and F_5 become modified by the pion pole terms but the contribution from the F_4 is negligible. The contribution from F_5 is negative (pion pole terms make it smaller) and its absolute values decreases quickly with Q^2 (see Fig. 4). Therefore, it reduces the differential cross section in low Q^2. In the case of antineutrino scattering the F_3 terms contributes with the negative sign. The antineutrino cross section is smaller and the F_5 induced reduction is more important.

There is another approximate way to include $m \neq 0$ effects: to apply exact expressions for $F_{1,2,3}$ as defined in the original RS model and to use Albright-Jarlskog relations (A-J) [7] for F_4 and F_5:

$$F_4 = 0, \qquad xF_5 = F_2 \tag{3}$$

We checked numerically that $xF_5 = F_2$ holds with an accuracy of 20% for $Q^2 \sim 0.05$ GeV^2, 10% for $Q^2 \sim 0.1$ GeV^2 and 1% for $Q^2 \sim 0.5$ GeV^2. The ratio $xF_5/F_2(W)$ depends very weakly on the hadronic invariant mass.

In Fig. 5 we present the similar plots as in Fig. 1. We see that for neutrino scattering the approximation based on the A-J relation is a good one but in antineutrino case the total cross section is underestimated. In Fig. 6 we see that at $E = 700$ MeV the approximation we discuss reconstructs well the $d\sigma/dQ^2$ in the case of neutrino reaction. For antineutrino scattering the approximation based on the A-J relations fails to reproduce both $d\sigma/dQ^2$ and its shape.

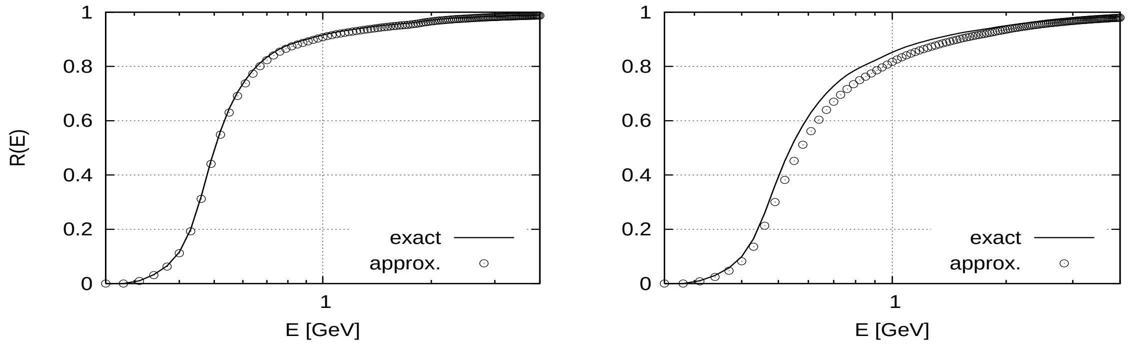

FIG. 5: Reduction of the total cross sections for reactions $\nu n \to \mu^- p \pi^0$ (left figure) and $\overline{\nu} p \to \mu^+ n \pi^0$ (right figure) due to $m \neq 0$. Two computations are compared: exact computations with pion pole contribution included (solid line) and the approximation based on Albright-Jarlskog relations (circles). The cut on the invariant hadronic mass $W < 2$ GeV is imposed.

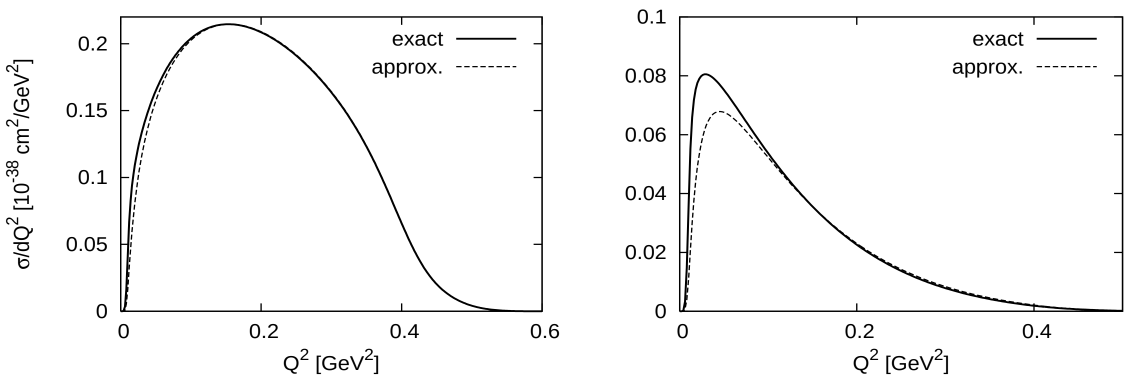

FIG. 6: Differential cross section for reactions $\nu n \to \mu^- \pi^0 p$ (left figure) and $\overline{\nu} p \to \mu^+ \pi^0 n$ (right figure) for E=700 MeV. Cross sections are obtained in exact computation (solid lines) and in approximation with F_4 and F_5 calculated with Albright-Jarlskog relations (dashed line).

The conclusions of our investigation is that (in the case of antineutrino reactions) it is improper to use RS model without pion pole terms. The approximation based on the A-J relation is satisfactory only for neutrino scattering and not in the antineutrino case.

After this paper was completed we learned about the work of Ch. Berger and L. Sehgal [9] in which the same problem is discussed. The authors of Ref. [9] use the formalism developed in [6] and in their presentation focus on modifications of the cross sections at small scattering angles.

APPENDIX A

Incoming neutrino and outgoing charged lepton 4-momenta are denoted as k^μ and k'^μ. Similarly, p^μ and p'^μ are target nucleon and outgoing resonance 4-momenta. 4-momentum transfer is $q^\mu \equiv k^\mu - k'^\mu = p'^\mu - p^\mu$, $Q^2 \equiv -q_\mu q^\mu$, $k^\mu = (E, \mathbf{k})$. In the Lab frame the axis orientation is chosen so that $q^\mu = (\nu, 0, 0, q)$. M denotes the nucleon's and M_R the resonance mass, W is the invariant hadronic mass of the final state, charged lepton mass is denoted by m. Explicit computations are done in the resonance rest frame and then the 4-vectors are labeled by subscripts $_{res}$.

In the RS model the differential cross section for neutrino/antineutrino- resonance production are expressed by matrix elements of $\mathcal{J}_0$, $\mathcal{J}_3$, $\mathcal{J}_+ \equiv -\frac{1}{\sqrt{2}}\left(\mathcal{J}_1 + i\mathcal{J}_2\right)$, $\mathcal{J}_- \equiv \frac{1}{\sqrt{2}}\left(\mathcal{J}_1 - i\mathcal{J}_2\right)$:

$$\frac{d^2\sigma}{d\nu dQ^2} = \frac{G^2\cos^2\theta_C}{4\pi E^2}(2\pi)^6\overline{\sum_{s,s'}}\frac{E_{p,res}}{M}\delta(W-M_R)\times$$
$$\Big\{D_0\left|\langle p'_{res},s'|\,\mathcal{J}_0\,|p_{res},s\rangle\right|^2 + D_3\left|\langle p'_{res},s'|\,\mathcal{J}_3\,|p_{res},s\rangle\right|^2 - 2D_{03}\mathrm{Re}\left(\langle p'_{res},s'|\,\mathcal{J}_0\,|p_{res},s\rangle\,\langle p_{res},s|\,\mathcal{J}_3\,|p'_{res},s'\rangle\right)$$
$$+(A\mp B)\left|\langle p'_{res},s'|\,\mathcal{J}_+\,|p_{res},s\rangle\right|^2 + (A\pm B)\left|\langle p'_{res},s'|\,\mathcal{J}_-\,|p_{res},s\rangle\right|^2\Big\} \tag{A1}$$

where $\mp$ refers to neutrino/antineutrino.

$$D_0 = 2E_{res}^2 - 2\nu_{res}E_{res} + k_\mu q^\mu, \quad D_3 = 2(k_{res}^3)^2 - 2q_{res}k_{res}^3 - k_\mu q^\mu, \quad D_{03} = \nu_{res}k_{res}^3 + q_{res}E_{res} - 2E_{res}k_{res}^3$$
$$A = |k_+^{res}|^2 - k_\mu q^\mu, \quad B = \left(q_{res}E_{res} - \nu_{res}k_{res}^3\right)$$

and also

$$k_{res}^3 = \frac{m^2+Q^2+2E_{res}\nu_{res}}{2q_{res}}, \quad |k_+^{res}|^2 = E_{res}^2 - (k_{res}^3)^2. \tag{A2}$$

In the limit $m\to 0$ the cross section (A1) is expressed in terms of matrix elements of $\mathcal{J}_-$, $\mathcal{J}_+$ and $\mathcal{J}_{\underline{0}} \equiv \mathcal{J}_0 + \frac{\nu_{res}}{q_{res}}\mathcal{J}_3$. Their values for 18 resonances are listed in Tab. II of Ref. [2]. In our computation we use the normalization from our previous paper [8]. In practice, functions S, B and C of this paper differ by the factor $1/2W$ with respect to analogical functions introduced in [2]. ($S_{this\ paper} = 2WS_{RS}$,... etc.).

Matrix elements of $\mathcal{J}_0$ and $\mathcal{J}_3$ can be obtained because in the RS model they have the same operational structure as $\mathcal{J}_{\underline{0}}$:

$$\mathcal{J}_{\underline{0}}^V(S) = 9\tau_a^+ S e^{-\lambda a^{3\dagger}}, \quad \mathcal{J}_{\underline{0}}^A(B,C) = -9\tau_a^+ e^{-\lambda a^{3\dagger}}\left(C\sigma_a^3 + B\vec{\sigma}_a\cdot\vec{a}^\dagger\right), \quad S = \frac{Q^2}{q_{res}^2}\frac{3WM-Q^2-M^2}{3W^2}G_V \tag{A3}$$

and all we need to do is to make substitutions:

$$\mathcal{J}_0^V = \mathcal{J}_{\underline{0}}^V\left(S\to S_0 = \frac{q_{res}^2}{Q^2}S\right), \quad \mathcal{J}_3^V = \mathcal{J}_{\underline{0}}^V\left(S\to S_3 = -\frac{\nu_{res}q_{res}}{Q^2}S\right) \tag{A4}$$

and

$$\mathcal{J}_0^A = \mathcal{J}_{\underline{0}}^A\left(B\to B_0, C\to C_0\right), \quad \mathcal{J}_3^A = \mathcal{J}_{\underline{0}}^A\left(B\to B_3, C\to C_3\right). \tag{A5}$$

with

$$B_0 = G_A Z\frac{2}{3}\sqrt{\frac{\Omega}{2}}, \quad C_0 = G_A Z\frac{Mq}{W}\left(\frac{1}{3} + \frac{W^2-Q^2-M^2}{(W+M)^2+Q^2}\right), \tag{A6}$$
$$B_3 = G_A Z\frac{4Mq}{3\left((W+M)^2+Q^2\right)}\sqrt{\frac{\Omega}{2}}, \tag{A7}$$
$$C_3 = G_A Z\left(\frac{3W^2+Q^2+M^2}{6} - \frac{2W}{(W+M)^2+Q^2}\left(q^2\frac{M^2}{W^2} + \frac{N}{3}\Omega\right),\right) \tag{A8}$$

where $\lambda = \sqrt{\frac{2}{\Omega}}q_{res}$, $Z = 0.7602$, Ω =1.05 GeV^2 is determined from the Regge slope of baryon trajectories.

In order to include pion pole terms we calculate:

$$q_{res}^\mu\mathcal{J}_\mu^A = \mathcal{J}_{\underline{0}}^A\left(B\to B_D \equiv \nu_{res}B_0 + q_{res}B_3, \quad C\to C_D \equiv \nu_{res}C_0 + q_{res}C_3.\right). \tag{A9}$$

and the final expressions for the axial current are:

$$\mathcal{J}_0^{A,mod} = \mathcal{J}_{\underline{0}}\left(B\to B_0 + \nu_{res}\frac{B_D}{m_\pi^2+Q^2}, \quad C\to C_0 + \nu_{res}\frac{C_D}{m_\pi^2+Q^2}\right), \tag{A10}$$
$$\mathcal{J}_3^{A,mod} = \mathcal{J}_{\underline{0}}\left(B\to B_3 - q_{res}\frac{B_D}{m_\pi^2+Q^2}, \quad C\to C_3 - q_{res}\frac{C_D}{m_\pi^2+Q^2}\right). \tag{A11}$$

APPENDIX B: STRUCTURE FUNCTIONS

The cross section for neutrino/antineutrino-nucleon scattering has the form:

$$\begin{aligned}\frac{d^2\sigma}{d\nu dQ^2} &= \frac{G^2\cos^2\theta_C}{4\pi E^2}\left\{(Q^2+m^2)\frac{F_1}{M}+\left(2E(E-\nu)-\frac{m^2+Q^2}{2}\right)\frac{F_2}{\nu}\right.\\ &\left.\pm\left(EQ^2-\frac{\nu}{2}(m^2+Q^2)\right)\frac{F_3}{\nu M}+\frac{m^2}{2}\left(Q^2+m^2\right)\frac{F_4}{\nu M^2}-\frac{m^2E}{\nu M}F_5\right\}. \qquad \text{(B1)}\end{aligned}$$

where

$$F_1 = (2\pi)^6\delta(W-M_R)\frac{M}{2}\overline{\sum_{s,s'}}\left\{\left|\langle p'_{res},s'|\,\mathcal{J}_-\,|p_{res},s\rangle\right|^2+\left|\langle p'_{res},s'|\,\mathcal{J}_+\,|p_{res},s\rangle\right|^2\right\}\frac{E_{p,res}}{M}, \qquad \text{(B2)}$$

$$\begin{aligned}F_2 &= (2\pi)^6\delta(W-M_R)\frac{E_{p,res}}{M}\frac{\nu Q^2}{2q^2}\\ &\overline{\sum_{s,s'}}\left\{\frac{2q_{res}^2}{Q^2}\left|\langle p'_{res},s'|\,\mathcal{J}_{\underline{0}}\,|p_{res},s\rangle\right|^2+\left|\langle p'_{res},s'|\,\mathcal{J}_-\,|p_{res},s\rangle\right|^2+\left|\langle p'_{res},s'|\,\mathcal{J}_+\,|p_{res},s\rangle\right|^2\right\}, \qquad \text{(B3)}\end{aligned}$$

$$F_3 = (2\pi)^6\frac{\nu M}{q}\delta(W-M_R)\overline{\sum_{s,s'}}\left\{\left|\langle p'_{res},s'|\,\mathcal{J}_-\,|p_{res},s\rangle\right|^2-\left|\langle p'_{res},s'|\,\mathcal{J}_+\,|p_{res},s\rangle\right|^2\right\}\frac{E_{p,res}}{M}, \qquad \text{(B4)}$$

$$\begin{aligned}F_4 &= (2\pi)^6\delta(W-M_R)\frac{E_{p,res}}{M}\frac{\nu}{q^2}\overline{\sum_{s,s'}}\left[\left|\langle p'_{res},s'|\,q_{res}\mathcal{J}_{\underline{0}}-W\mathcal{J}_3\,|p_{res},s\rangle\right|^2\right.\\ &\left.-\frac{M^2}{2}\left(\left|\langle p'_{res},s'|\,\mathcal{J}_-\,|p_{res},s\rangle\right|^2+\left|\langle p'_{res},s'|\,\mathcal{J}_+\,|p_{res},s\rangle\right|^2\right)\right], \qquad \text{(B5)}\end{aligned}$$

$$\begin{aligned}F_5 &= (2\pi)^6\delta(W-M_R)\frac{E_{p,res}}{M}\frac{\nu}{2q^2}\overline{\sum_{s,s'}}\left[\left|\langle p'_{res},s'|\,2q_{res}\mathcal{J}_{\underline{0}}-W\mathcal{J}_3\,|p_{res},s\rangle\right|^2-W^2\left|\langle p'_{res},s'|\,\mathcal{J}_3\,|p_{res},s\rangle\right|^2\right.\\ &\left.-\left(M^2-Q^2-W^2\right)\left(\left|\langle p'_{res},s'|\,\mathcal{J}_-\,|p_{res},s\rangle\right|^2+\left|\langle p'_{res},s'|\,\mathcal{J}_+\,|p_{res},s\rangle\right|^2\right)\right]. \qquad \text{(B6)}\end{aligned}$$

In order to calculate F_1, F_2 and F_3 it is enough to know matrix elements of $\mathcal{J}_\pm$ and $\mathcal{J}_{\underline{0}}\equiv\mathcal{J}_0+\frac{\nu_{res}}{q_{res}}\mathcal{J}_3$ and they are provided in the original RS model. Calculation of F_4, F_5 requires an additional knowledge of matrix elements of $\mathcal{J}_3$. In the limit $m\to 0$ the contribution from F_4 and F_5 to cross section vanishes.

F_1, F_2 and F_3 do not depend on pion pole terms because they do not modify $\mathcal{J}_\pm$ and $\mathcal{J}_{\underline{0}}$.

ACKNOWLEDGEMENTS

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